

STATIC MASS SCALES IN HOT GAUGE THEORIES

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The static sectors of the electroweak Standard Model and QCD at finite temperatures are described by 3d SU(N) Higgs models with scalars in the fundamental and adjoint representation, respectively. I summarize the non-perturbative physics of these theories like mass spectrum, string tension, screening of the static potential and non-perturbative corrections to the Debye mass, as obtained from recent lattice simulations. It is observed that in the 3d theory corresponding to the purely magnetic sector there is a larger hierarchy between different quantities than between the electric ($\sim gT$) and magnetic ($\sim g^2T$) sectors of finite T gauge theories, which are not generally well separated.

1 Introduction

Gauge theories in 3d play an important role for high temperature particle physics, since they constitute the Matsubara zero mode sector of finite temperature quantum field theories in the imaginary time formalism. In the framework of dimensional reduction they emerge as effective theories describing all static properties and the equilibrium thermodynamics of the original 4d finite T theory. Hence, any static physical quantity of a finite T theory must have a corresponding quantity in the effective 3d theory, with which it agrees up to a (perturbative) part due to the non-zero Matsubara modes. For example, a static screening length in finite T field theory is defined as the exponential decay of some *spatial* correlation function. In the 3d theory, the direction of the correlation may be taken to be (euclidean) time, and hence the same quantity appears in the spectrum or some other physical property of the (2+1)d theory. While dimensional reduction is a perturbative procedure, the resulting effective theories for the symmetric phase of the electroweak Standard Model and hot QCD are 3d SU(N) gauge + fundamental and adjoint Higgs models, respectively, in their confining phases and hence entirely non-perturbative.

Here I discuss the physical properties of the 3d SU(2) Higgs models with fundamental and adjoint scalar fields. The scale is set by the dimensionful gauge coupling $g_3^2 = g^2T$. Further, the physics of both models is fixed by two parameters x and y , which are dimensionless ratios of the scalar coupling and bare mass with the gauge coupling, respectively. For effective high T theories, x, y are fixed by dimensional reduction¹ as functions of T and the Higgs mass (fundamental) or of T alone (adjoint). Both models have confinement and Higgs regions in their phase diagram, which are separated by a first order phase transition for small x , but analytically connected² for large x .

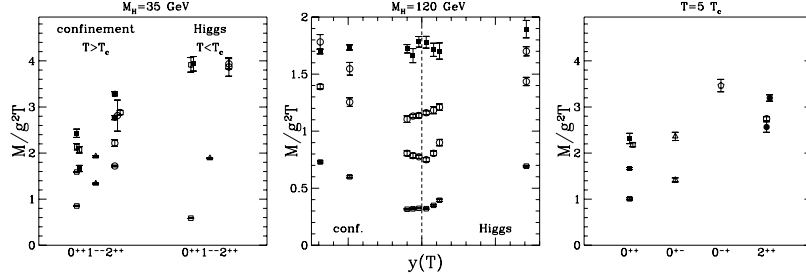


Figure 1: Left: Spectrum of static screening masses for the SU(2) + fundamental Higgs model for a light Higgs ($x = 0.0239$). Middle: 0^{++} spectrum as function of T for heavy Higgs ($x = 0.274$). Right: Spectrum for SU(2) + adjoint Higgs, corresponding to $T \approx 5T_c$ ($x = 0.104$, $y = 0.242$) in dim.red. SU(2) QCD.

2 Mass spectrum

The physical properties of gauge theories are encoded in gauge-invariant n -point functions. In particular, the mass spectrum is computed from the exponential fall-off of two-point correlation functions

$$\lim_{|x-y| \rightarrow \infty} \langle \phi^\dagger(x) \phi(y) \rangle \sim e^{-M|x-y|}, \quad (1)$$

where ϕ generically denotes some gauge-invariant operator with quantum numbers J^{PC} . The results of lattice calculations³ of the lowest states of the spectra at various points in the parameter space of our models are displayed in Fig. 1.

First, consider the electroweak model for small Higgs mass, when confinement and Higgs region are separated by a first order phase transition. In the Higgs region we see the familiar and perturbatively calculable Higgs and W-boson in the 0^{++} and 1^{--} channels, respectively. There is a large gap to the higher excitations which are scattering states. In the confinement region, in contrast, there is a dense spectrum of bound states in all channels, very much resembling the situation in QCD. Open symbols denote bound states of scalar fields, whereas full symbols represent glueballs. At a more realistic large Higgs mass, the situation in the confinement region (above the phase transition) is repeated, but the picture in the Higgs region has changed to a similarly dense spectrum. No phase transition separates the two regimes and the mass spectrum can be continuously connected. Nevertheless, the two regimes have different dynamics, e.g. no confining gauge string is formed between static sources in the Higgs regime right of the dashed line, whereas the properties of the glueball spectrum are entirely insensitive to variations of the scalar parameters to the left of it. The line marks the center of a rapid but smooth transition between these regimes.

	$a\sqrt{\sigma}$	am_G	am_G^*
pure gauge	0.1622(4)	0.755(7)	1.09(2)
fundamental Higgs	0.1575(5)	0.74(3)	1.08(4)
adjoint Higgs	0.1560(70)	0.74(1)	1.03(5)

Table 1: Comparison of string tension, the mass of the 0^{++} glueball and its first excitation in lattice units as measured for equal lattice spacing, $\beta = 4/(ag_3^2) = 9$.

Next, consider the SU(2) adjoint Higgs model, corresponding to dimensionally reduced ¹ SU(2) QCD with $N_f = 0$. Fig. 1 right shows preliminary results ⁵ for some low lying states at a point corresponding to $T \approx 5T_c$. In contrast to the fundamental Higgs model one can also define gauge-invariant operators odd under charge conjugation. Otherwise the situation is completely analogous, with again a repetition of the pure gauge glueball spectrum, denoted by the full symbols, and bound states of adjoint scalars, and similarly little dependence of the gauge part on the scalar parameters.

The properties of the pure gauge sector of the two types of Higgs model in their confinement phases are compared with those of pure gauge theory ⁴ in Table 1. The masses of the glueballs as well as the string tension in the linear part of the potential agree remarkably well between the three models, demonstrating that the dynamics of the gauge degrees of freedom is almost entirely insensitive to the presence of matter fields.

3 Static potential and screening

Another quantity determining the physical properties of confining theories is the potential energy of static colour sources, which is calculated from the exponential decay of large Wilson loops,

$$V(r) = - \lim_{t \rightarrow \infty} \frac{1}{t} \ln W(r, t). \quad (2)$$

As in four dimensions, a string of colour flux connects the sources, both in fundamental and adjoint representation, leading to a potential rising linearly with their separation. For the fundamental potential in pure gauge theory, this linear rise continues to infinity. If fundamental matter fields are present as in the Higgs model, the string breaks at some scale r_b , when its energy is large enough to produce a pair of scalars. This results in a saturation of the potential at a constant value corresponding to the energy of two static-light mesons which are formed after string breaking, $V(r \rightarrow \infty) = \text{const.}$

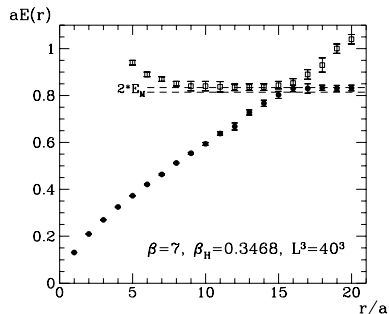


Figure 2: Static potential between fundamental sources in the confinement region of the $SU(2) + \text{fundamental Higgs}$ model.

Considering adjoint sources instead, string breaking occurs already in pure gauge theory, because the adjoint string can couple to pair-produced gluons.

The string breaking scale r_b is a physical quantity characterizing the range of the confining force. Its size depends on the string tension and the mass of the dynamical particles that have to be produced to break the string. For the fundamental potential in the $SU(2)$ Higgs model it thus depends on the bare scalar mass in the Lagrangian. In a recent lattice simulation⁶ r_b has been computed for the same parameter values as the light Higgs confining spectrum (cf. Fig. 1) by extracting it from the turnover of the potential as shown in Fig. 2. The continuum extrapolation of those results gives $r_b g_3^2 \approx 8.5$, $r_b^{-1} \approx 0.12 g_3^2$. For comparison, the lightest scalar bound state from Fig. 1 is $m_S = 0.839(15) g_3^2$, and the lightest glueball $m_G = 1.60(4) g_3^2$. On the other hand, considering the adjoint potential in pure gauge theory, there is no bare mass in the Lagrangian allowing to tune the mass of the constituents, and hence r_b is a purely dynamical quantity of the theory. In this case the continuum result is⁶ $r_b g_3^2 = 6.50(94)$, or $r_b m_G = 10.3 \pm 1.5$. In other words, the 3d pure gauge theory contains a mass scale r_b^{-1} which is by an order of magnitude smaller than the mass of the lightest physical state.

4 The Debye mass

An important concept in the phenomenology of high temperature QCD is the static electric screening mass, or the Debye mass m_D . Although its leading order contribution is perturbative, it couples to the 3d magnetic sector in next-to-leading order, and hence requires a non-perturbative treatment as well. The

Debye mass can be expanded as⁷

$$m_D = m_D^{\text{LO}} + \frac{Ng^2T}{4\pi} \ln \frac{m_D^{\text{LO}}}{g^2T} + c_N g^2T + \mathcal{O}(g^3T), \quad (3)$$

where $m_D^{\text{LO}} = (N/3 + N_f/6)^{1/2}gT$ and N_f is the number of flavours. The logarithmic part of the $\mathcal{O}(g^2)$ correction can be extracted perturbatively⁷, but c_N and the higher terms are non-perturbative. To allow for a lattice determination, a non-perturbative definition was formulated⁸ employing the SU(N) adjoint Higgs model as the dimensionally reduced effective theory. By integrating out the heavy adjoint Higgs this is further reduced to the pure SU(N) theory in 3d. The coefficient c_N can then be determined from the exponential fall-off of an adjoint Wilson line $U_{ab}^{\text{adj}}(x, y)$ with appropriately chosen adjoint charge operators at the ends, for example

$$G_F(x, y) = \langle F_{ij}^a(x) U_{ab}^{\text{adj}}(x, y) F_{kl}^b(y) \rangle. \quad (4)$$

From its measurement in a lattice simulation of 3d pure SU(2) one finds the complete non-perturbative $\mathcal{O}(g^2T)$ corrections to the Debye mass with high precision to be⁹ $c_2 = 1.06(4)$. Comparing this correction with the ($N_f = 0$) leading term, one finds $c_2 g^2T/m_D^{\text{LO}} = 1.3g$ which is close to one even for couplings smaller than one.

5 Summary

Three-dimensional gauge theories exhibit a rich structure of physical mass scales. Although all of them are necessarily $\sim \mathcal{O}(g_3^2 = g^2T)$, the coefficients vary by more than an order of magnitude. For finite temperature field theory this means that non-perturbatively the hierarchy of scales in the purely magnetic sector is larger than that between the electric and the magnetic sector, which are not generally well separated for realistic couplings.

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